SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 1, WS 2017/18
Sheet 10

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1. Which of the following series are convergent?

(a) $\displaystyle\sum_{r=1}^{\infty} \frac{r^3+4r+3}{\sqrt{r^{10}+r^7}}$

(b) $\displaystyle\sum_{r=1}^{\infty} \frac{r^3+4r+3}{\sqrt{r^8+3r^7}}$

(c) $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(1+1/r)^r}$

(d) $\displaystyle\sum_{r=1}^{\infty} \frac{r^4+1}{2^r}$
[Hint: $r^4+1 \leq (\frac{3}{2})^r$ for large r]

(e) $\displaystyle\sum_{r=1}^{\infty} \frac{r+2^r}{r2^r}$
[Hint: $r+2^r \geq 2^r$]

(f) $\displaystyle\sum_{r=1}^{\infty} \frac{r!}{r^r}$
[Hint: ratio test]

(g) $\displaystyle\sum_{r=1}^{\infty} \sin\frac{1}{r}$
[Hint: $\sin x \geq \frac{1}{2}x$ for small x]

(h) $\displaystyle\sum_{r=1}^{\infty} \frac{1}{r}\sin\frac{1}{r}$
[Hint: $\sin x \leq x$]

(i) $\displaystyle\sum_{r=2}^{\infty} \frac{1}{r\log r}$
[Hint: $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}(\log(\log x)) = \frac{1}{x\log x}$]

(j) $\displaystyle\sum_{r=2}^{\infty} \frac{1}{r^2\log r}$
[Hint: $\log r > 1$ for large r]

2. (a) How many n-digit natural numbers without the digit 9 are there?

(b) Prove that the sum of the reciprocal values of the n-digit natural numbers without the digit 9 is less than or equal to $8(\frac{9}{10})^{n-1}$.

(c) Prove that the series obtained from the harmonic series by removing those summands with the digit 9 in their denominator is convergent.

3. Give rigorous formulations of the following statements.

(i) $f(x) \to \infty$ for $x \to \infty$

(ii) $f(x) \to -\infty$ for $x \to \infty$

(iii) $f(x) \to \infty$ for $x \to -\infty$

(iv) $f(x) \to -\infty$ for $x \to -\infty$

(v) $f(x) \to \infty$ for $x \to a$

(vi) $f(x) \to -\infty$ for $x \to a$