



Mathematics for Computer Scientists 1, WS 2018/19
Sheet 4

1. Determine the real numbers x for which the following inequalities hold.

(a) $\frac{4x - 5}{x^2 - 1} < 5$

(d) $\left| \frac{(x - 1)(2x - 3)}{x(x - 5)} \right| > 1$

(b) $\frac{5}{5x - 1} < \frac{2}{2x + 1}$

(e) $\log \left(\frac{2 - x}{12 + 4x} \right) > 0$

(c) $\frac{3x + 2}{2x + 3} < \frac{x}{x + 1}$

(f) $e^x > 3^{x^2}$

2. Sketch the subsets

$$A_1 = \left\{ (x, y) : 3x + 2y \leq 6, x - y \leq 2, x \leq 1 \right\},$$

$$A_2 = \left\{ (x, y) : |y| \leq \frac{\sqrt{5}}{2}, |y - \sqrt{5}x| \leq \sqrt{5}, |y + \sqrt{5}x| \leq \sqrt{5} \right\}$$

of the (x, y) coordinate plane.

3. Sketch the set $A \cap B$, where

$$A = \{z \in \mathbb{C} : |z - 2 - 3i| < |z + 4 - 5i|\},$$

$$B = \{z \in \mathbb{C} : 0 \leq \arg(z + 3 - 4i) < \pi/4\}.$$

4. Find all complex solutions to the following equations.

(a) $3z^2 + z = 1$

(g) $(z^2 - 1)^3 = 8z^3$

(b) $z^2 - (3 + i)z + 4 + 3i = 0$

(h) $z^6 - 3iz^3 - 2 = 0$

(c) $\sinh z = i$

(i) $z^3 + 2z^2 + 2z = 0$

(d) $z^2 + 2\bar{z}^2 + z - \bar{z} + 9 = 0$

(j) $z^3 - (3 + i)z^2 + (2 + 3i)z - 2i = 0$

(e) $z^4 - 4z^2 + 16 = 0$

(k) $e^z = e^{iz}$

(f) $z^4 + 1 = 0$

(l) $e^{2z} + ie^z + 1 = 0$

5. Compute $(4\sqrt{3} - 4i)^{88}$. [Hint: use de Moivre's theorem.]