SAARLAND UNIVERSITY
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Mathematics for Computer Scientists 1, WS 2017/18
Sheet 2

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1. Let $G = \{1, 2, 3, 4\}$ and define relations R_1, R_2, R_3 on G by

$$\begin{aligned} R_1 &= \{(1,2),(2,1)\}, \\ R_2 &= \{(1,1),(2,2),(3,3),(4,4)\}, \\ R_3 &= \{(1,1),(1,3),(2,2),(3,1),(3,3),(4,4)\}. \end{aligned}$$

Are these relations reflexive, complete, symmetric, asymmetric, antisymmetric and/or transitive?

2. Define relations $\sim_a$, $\sim_a$, $\sim_c$ on $\mathbb{Z}$ by

$$\begin{aligned} x \sim_a y \qquad &\Leftrightarrow \qquad x \neq y, \\ x \sim_b y \qquad &\Leftrightarrow \qquad x + y \text{ is an even number}, \\ x \sim_c y \qquad &\Leftrightarrow \qquad x \geq y^2. \end{aligned}$$

Are these relations reflexive, complete, symmetric, asymmetric, antisymmetric and/or transitive?

3. Define a relation $\sim$ on $\mathbb{N}_0 \times \mathbb{N}_0$ by

$$(p, n) \sim (q, m) \qquad \Leftrightarrow \qquad p + m = q + n.$$

(a) Show that $\sim$ is an equivalence relation on $\mathbb{N}_0 \times \mathbb{N}_0$.

(b) Show that

$$(p, n) \sim (k + p, k + n)$$

for all $k \in \mathbb{N}_0$.

(c) Denote the equivalence class $[(k, 0)]$ by $\mathbf{k}$ and define the 'sum' of two equivalence classes by the formula

$$[(p, n)] \boldsymbol{+} [(q, m)] = [(p + q, n + m)].$$

Determine the equivalence class $\mathbf{-k}$ with the property that

$$\mathbf{-k} \boldsymbol{+} \mathbf{k} = \mathbf{0}.$$

[You may assume that '$\boldsymbol{+}$' is well defined.]

4. Let M be a non-empty set and define a relation $\preceq$ on the power set $P(M)$ of M by

$$A \preceq B \qquad \Leftrightarrow \qquad A \subseteq B.$$

Show that this relation is a partial order. When is it a total order?