SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 1, WS 2017/18
Sheet 1

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1. Consider the subsets

$$\begin{aligned} A &= \{(x, y) : (x-3)^2 + y^2 \leq 1\}, \\ B &= \{(x, y) : (x+2)^2 + y^2 \leq 1\}, \\ C &= \{(x, y) : |y-1| \leq 1\}, \\ D &= \{(x, y) : |x| \leq 2, |y-2| \leq 2\}, \\ E &= \{(x, y) : -4 < x < 3\} \end{aligned}$$

of the coordinate plane P.

Sketch the sets A, B, C, D, E and $A \cup B \cup C \cup D$, $(A \cup B \cup C \cup D) \cap E$, $B \cap C$, $B \cap D$, $(B \cap C) \setminus (B \cap D)$.

2. Let A, B, X und Y be subsets of a universal set U.

(a) Prove *from the axioms of set theory* that $X \cup U = U$ and $X \cap \emptyset = \emptyset$.

(b) Prove *from the axioms of set theory* that $(A \cup B) \cup (\overline{A} \cap \overline{B}) = U$ and $(A \cup B) \cap (\overline{A} \cap \overline{B}) = \emptyset$.

(c) Let $X \cup Y = U$ and $X \cap Y = \emptyset$. Prove *from the axioms of set theory* that $Y = \overline{X}$.

(d) Deduce de Morgan's rule

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

and the involution identity

$$\overline{\overline{A}} = A.$$

3. Let L, M and N be arbitrary sets. Prove or give a counterexample to each of the following statements.

(i) $(L \backslash M) \backslash N = L \backslash (M \cup N)$,

(ii) $L \backslash (M \cap N) = (L \backslash M) \cap (L \backslash N)$,

(iii) $L \backslash (M \cup N) = (L \backslash M) \cup (L \backslash N)$.

4. Which of the following functions $f : \mathbb{R} \to \mathbb{R}$ is injective, surjective, bijective? (Justify your answers.) Compute $f([-1,1])$ and $f^{-1}([-1,1])$ in each case.

a) $f_1(x) = \begin{cases} x-1 & \text{for } x \geq 0, \\ x+1 & \text{for } x < 0. \end{cases}$

b) $f_2(x) = \begin{cases} -x-1 & \text{for } x \geq 0, \\ -x+1 & \text{for } x < 0. \end{cases}$

c) $f_3(x) = \begin{cases} x^2 & \text{for } x \geq 0, \\ x^3 & \text{for } x < 0. \end{cases}$

5. Consider the sets

$$\begin{array}{llll} S_1 = \{\{\emptyset\}, \{A\}, A\}, & S_2 = A, & S_3 = \{A\}, & S_4 = \{A, \{A\}\}, \\ S_5 = \emptyset, & S_6 = \{\emptyset\}, & S_7 = \{\{\emptyset\}\}, & S_8 = \{\emptyset, \{\emptyset\}\}. \end{array}$$

(i) Which of the sets $S_1, ..., S_8$ is an element of S_1?

(ii) Which of the sets $S_1, ..., S_8$ is a subset of S_1?

(iii) Which of the sets $S_1, ..., S_8$ is an element of S_8?

(iv) Which of the sets $S_1, ..., S_8$ is a subset of S_8?