SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 2, SS 2018
Sheet 11

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1. Which of the matrices

$$\begin{pmatrix} -3 & 4 \\ -1 & 1 \end{pmatrix}, \qquad \begin{pmatrix} -3 & 1 \\ -2 & -1 \end{pmatrix}, \qquad \begin{pmatrix} -3 & 3 \\ 2 & -2 \end{pmatrix}, \qquad \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$$

are diagonalisable over $\mathbb{Q}$, $\mathbb{R}$ or $\mathbb{C}$?

2. Let

$$A = \begin{pmatrix} -5 & 3 \\ 6 & -2 \end{pmatrix}.$$

(a) Find a real, invertible matrix P such that $P^{-1}AP$ is diagonal.

(b) Find a real, invertible matrix B such that $B^3 = A$.

3. The *Legendre polynomials* are defined by the formulae

$$P_n(x) = \frac{1}{2^n n!}\frac{\mathrm{d}^n}{\mathrm{d}x^n}\big((1-x^2)^n\big), \qquad n = 1, 2, 3, \ldots.$$

Show that $\{P_n\}_{n=1}^{\infty}$ is an orthogonal set in $\mathcal{P}(\mathbb{R})$ i.e. $\langle P_n, P_m\rangle = 0$ for $n \neq m$, where

$$\langle p_1, p_2\rangle = \int_{-1}^{1} p_1(x)p_2(x)\,\mathrm{d}x, \qquad p_1, p_2 \in \mathcal{P}(\mathbb{R}).$$

[Hint: derive the formula

$$\frac{\mathrm{d}}{\mathrm{d}x}\big((1-x^2)P_n(x)\big) + n(n+1)P_n(x) = 0$$

and consider the integral

$$\int_0^1 \left(P_m(x)\frac{\mathrm{d}}{\mathrm{d}x}\big((1-x^2)P_n(x)\big) - P_n(x)\frac{\mathrm{d}}{\mathrm{d}x}\big((1-x^2)P_m(x)\big)\right)\mathrm{d}x.$$]